Among the classic works on geometry in foreign languages *La Thèorie Génerale des Surfaces* by Darboux, published by Gauthier-Villars, Paris, France, is one of the most noted. This consists of four large volumes, and was published from 1887 to 1896. The second edition of Bianchi's *Legioni di Geometria Differenziale*, 1902-03, in two volumes of 523 and 594 pages, respectively, published by Spoerri, Pisa, Italy, is also very well known.

The preceding remarks relate separately to the three great fields of pure mathematics,—algebra, analysis, and geometry. A great deal of the most important mathematical literature does not limit itself to any one of these fields. Moreover, the student of mathematics needs outlook and independence as soon as he has sufficient knowledge to use these. That is, he needs books on books as well as thoughts on thoughts and ideas on ideas. Fortunately, such literature is growing very rapidly, although the bulk of it is in foreign languages. One of the most useful aids along this line in English is the Subject Index, Volume I, Pure Mathematics, of the Royal Society of London *Catalogue of Scientific Papers*, 1800-1900, published by the Cambridge, England, University Press, in 1908. This volume of about 700 pages is a subject index of the mathematical articles which appeared in 700 different serials during the nineteenth century, and is said to contain 38,748 entries.

Although this Index gives only the places where articles relating to different subjects were published it is of great value as one can often learn much in reference to what is probably new and what has been done by others from the various subjects to which articles relate and from the extent of these articles. The student of mathematics should never lose sight of the fact that good books are his tools and that it is almost as necessary for him to have the proper books as for the mechanic to have the proper tools. Only those who employ the best available tools for the work in hand have reason to expect notable success.

The Index mentioned above has been continued since 1900 by the annual mathematical volume in the *International Catalogue of Scientific Literature*, published for the International Council by the Royal Society of London. Each of the seventeen separate annual volumes of this catalogue contains both a subject and an author index, but it does not furnish more information about the article than the title conveys. The only extensive mathematical work which gives critical reviews of articles appearing from year to year is the excellent German publication entitled, *Jahrbuch über die Fortschritte der Mathematik*. About sixty different mathemations coöperate in providing reviews for this publication, which has appeared practically each year since 1871, and constitutes a most valuable source of information. The number of different articles reviewed at present is about three thousand per year and about one-third of a page is devoted, on an average, to each review.

As these reviewers are specialists of high scientific standing they have produced by their combined efforts a monumental work, which has had a most salutary influence on the development of mathematics, as it encour-

ages publications of high order and discourages research publications of little or no scientific value, as the latter publications are exposed to unfavorable reviews in this well known annual. It would be very desirable to have at least one more such annual, since even men of high scientific standing are not always free from prejudice. On the whole, it must be said that the reviews in the *Fortschritte* have been excellent and have more often dealt too leniently than too severely with the articles under consideration.

The most magnificent direct coöperation of mathematicians to produce a great work has been called into existence for the sake of completing the great German and French mathematical encyclopedias which are now in the course of publication by the firms of Teubner of Leipzig, Germany, and Gauthier-Villars of Paris, France. According to a recent circular more than 160 different mathematicians have been working on the German edition and more than 100 on the French. The vastness of the work may be inferred from the fact that the published parts of the German edition would fill more than a score of volumes of four hundred pages each although a large part of the field has not yet been covered; and, judging from the parts of the French that have appeared, this edition will be at least twice as large as the German. For instance, the number of pages in the published parts of Volume I of both of these two editions are as follows; the first number applying to the German edition. Fundamental principles of arithmetic, 27, 62; combinatory analysis and determinants, 19, 70; irrational numbers and convergence of infinite processes with real numbers, 100, 196; ordinary and higher complex numbers, 37, 140; infinite algorithms with complex numbers, 8, 20; theory of sets, 24, 42; finite discrete groups, 19, 85.

The object of this great work of reference is to give as completely as possible the fully established mathematical results and to exhibit by means of careful references the historical development of mathematical methods since the beginning of the nineteenth century. The work is not restricted to the so-called pure mathematics, but it includes applications to mechanics, physics, astronomy, geodesy, and various other technical subjects, so as to exhibit *in toto* the position occupied by mathematics in the present state of our civilization.

While such a vast work will naturally appeal to those interested in higher mathematics more than to those whose main interests are confined to the more elementary subjects, yet the latter will find much that is within their easy comprehension, especially in the introductory parts of arithmetic, geometry, and algebraic analysis. Moreover, it is of considerable value to read things once in a while which are not within one's easy comprehension. Great thoughts can sometimes be enjoyed even by those who cannot comprehend them completely, and a superficial view of the vastness of the developed parts of mathematics is much better than total ignorance and frequently awakens an interest in a particular field which appears to have received relatively too little attention. It may also serve to call attention to the im-

portant problem of mathematical transportation, or the utilization of the developments of one field in other fields, and the bringing together of facts whose similarity becomes striking through proximity. All the better libraries should be induced to subscribe for at least one of these great encyclopedias since such great works are of permanent value. Descriptive circulars can be obtained gratis by addressing the publishers as well as from some importers.

Among the less extensive works giving an outline of the main fields of mathematics which have been developed, the second edition of Pascal's *Repertorium der hoeheren Mathematik* takes the foremost place. The first two volumes, each covering more thon 500 pages, and costing about two dollars and a half, appeared in 1910, and the remaining two volumes are expected to appear towards the end of the present year. This work is also published by B. G. Teubner of Leipzig, Germany. In fact, this firm is now publishing more works on advanced mathematics than any other firm in the world.

Another very useful general work, which is, however, less modern in spirit, is Hagen's *Synopsis der hoeheren Mathematik*, consisting of three quarto volumes of about 400 pages each. This work was prepared in America when Hagen was Director of the Observatory of Georgetown College, but it was published in Germany by Dames, of Berlin. It may be of interest to observe that while the number of people who use the English language is very much larger than the number of those who use German, a few American mathematical writers have considered it best to publish their works in German. With the increase of scientific interest in China and Japan, where English is used much more than German or French, and with the growth of scientific interest among English speaking nations, it would appear that the English scientific works should soon command the most extensive market, and that more enterprising publishers of English scientific literature should come into existence.

There is a large number of other good general works on mathematics but the few that have been mentioned are among the very best, and they contain references to a large number of others. If any one should desire a cheaper guide through the mathematical literature but one which is fairly reliable and fairly extensive, he might be pleased with Mueller's *Fuehrer durch die mathematische Literature*, published by Teubner in 1909, and costing about two dollars.

In a very general way one may say that the German language contains the most along the lines of general literature, including the history of mathematics, as well as along algebraic and number theory lines. The French is richest along the line of analysis and comprehensive treatises on extensive fields of mathematics. The Italians are now ahead along geometrical lines, while the English excel in mathematical physics. It is hoped that the rapid recent advances in good English literature on various fields of pure

mathematics will have a good effect in creating a greater demand for such literature. Two recent American publications which may reasonably be expected to exert a good influence along this line are J. W. Young's *Fundamental Concepts of Algebra and Geometry*, and J. W. A. Young's *Monographs on Modern Mathematics*. These were published by the Macmillan Company, and by Longmans, Green and Company, respectively.

## NOTE ON THE BINOMIAL SERIES.

By K. OGURA in Sendai, Japan.

Euler proved the binomial theorem

$$F(y)=(1+x)^y=\sum_{n=0}^{\infty}\frac{y(y-1)\ldots(y-n+1)}{n!}x^n \qquad |x|<1,$$

provided that $y$ is a rational number. There are various proofs of the above formula when $y$ is irrational.

But it is hoped that the following proof will be found both simple and rigorous.

In the binomial series

$$F(y)=\sum_{n=0}^{\infty}\frac{y(y-1)\ldots(y-n+1)}{n.}x^n \qquad |x|<1,$$

put

$$f_n(y)=\frac{y(y-1)\ldots(y-n+1)}{1.2\ldots n}.$$

Let us denote by $G$ a finite positive number however great, and let us take $y$ so that $-G<y<G$.

(i) If $y>-1$, we can choose the positive numbers $k$ and $n$ so that $y+1<k<n$. Then

$$0<\frac{y+1}{k}<1, \quad 0<\frac{y+1}{k+1}<1, \ldots, 0<\frac{y+1}{n}<1, \ldots$$

and

$$|f_n(y)|=\left|(1-\frac{y+1}{1})(1-\frac{y+1}{2})\ldots(1-\frac{y+1}{k-1})\right|\times(1-\frac{y+1}{k})\ldots(1-\frac{y+1}{n}).$$

But since

$$\lim_{n=\infty} \sum_{\nu=1}^{n} \frac{y+1}{\nu} = +\infty,$$

we have

$$\prod_{n=k}^{\infty} \left(1 - \frac{y+1}{n}\right) = 0.$$

Hence

$$\lim_{n=\infty} |f_n(y)| = 0.$$

Therefore we can choose $n$ so that $|f_n(y)| < A$, $A$ being a fixed positive number.

(ii) If $y=-1$, then

$$|f_n(y)| = 1.$$

(iii) If $y<-1$, we can choose a positive number $p$, independent of $y$, so that $-y<1+p<G$. Then

$$|f_n(y)| < (1+\frac{p}{1})(1+\frac{p}{2})\ldots(1+\frac{p}{n}) < e^{p}.e^{\frac{1}{2}p}\ldots e^{p/n}$$

$$= e^{p(1+\frac{1}{2}+\ldots+1/n)} < e^{p(C+\log n)} = e^{pC}.n^{p},$$

$C$ being Euler-Mascheroni's constant.

If we put $A=e^{pC}$, then $|f_n(y)| < A.n^{p}$.

Hence, in general, there exists a positive number $X=1$ such that

$$|f_n(y)|\, X^n < An^{p}$$

for fixed positive $A$ and $p$.

Therefore, by a theorem due to Pringsheim,* it follows that the series $F(y)$ is a continuous function of $y$. $(-G<y<G)$.

Now if an irrational number $y$ be represented as the limit of the sequence $(y_1, y_2, \ldots, y_n, \ldots)$ whose terms are all rational, then we have

$$F(y) = \lim_{n=\infty} F(y_n) = \lim_{n=\infty} (1+x)^{y_n}.$$

But by the definition of the power

* Pringsheim, *Ber. München*, 27 (1897), p. 35; Bromwich, *Infinite Series*, 1908, p. 135.

$$\lim_{n=\infty}(1+x)^{y_n}=(1+x)^y.$$

Hence we have $F(y)=(1+x)^y$; that is,

$$(1+x)^y=1+\frac{y}{1!}x+\frac{y(y-1)}{2!}x^2+\ldots \quad |x|<1.$$

Thus the binomial theorem is proved for all finite values of $y$.

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## MOMENT OF INERTIA OF A RING CALCULATED BY AN ELEMENTARY METHOD.

By B. H. BROWN, Whitman College, Walla Walla, Washington.

The problem of finding the moment of inertia of a homogeneous circular ring, of unit density and of radius $R$ with cross-section $\pi r^2$, when referred to a diameter, appears to be quite generally avoided by our writers on Elementary Mechanics. The following solution may prove of interest.

Suppose the ring to be edgewise to the plane of the paper and bisected by the plane of the paper giving the two circles of the figure as cross-sections of the ring. Plane $PQ$ perpendicular to the paper bisects the ring and contains the axis of rotation, $PQ$ passing through $O$, the center of the ring. The moment of inertia of the central section containing the plane $PQ$ and of thickness $dy$ is given by

$$I_c=\frac{\pi}{4}(A^4-a^4)dy. \quad \text{But } A=R+(r^2-y^2)^{\frac{1}{2}} \text{ and } a=R-(r^2-y^2)^{\frac{1}{2}}.$$

$$\therefore I_c=\frac{\pi}{4}\{[R+(r^2-y^2)^{\frac{1}{2}}]^4-[R-(r^2-y^2)^{\frac{1}{2}}]^4\}dy$$

$$=2\pi R^3(r^2-y^2)^{\frac{1}{2}}dy+2\pi R(r^2-y^2)^{\frac{3}{2}}dy.$$

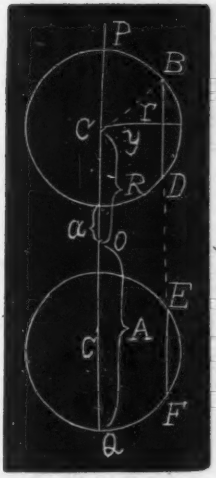
The moment of inertia for any other section as $BF$ parallel to $PQ$ at a distance $y$ from the axis is of course given by $I_y=I_c+my^2$.

The area $BDEF$ cut from the ring by plane $BF$ is equal to

$$\pi[R+(r^2-y^2)^{\frac{1}{2}}]^2-\pi[R-(r^2-y^2)^{\frac{1}{2}}]^2 \text{ or } 4\pi R(r^2-y^2)^{\frac{1}{2}},$$

and $m$ equals this quantity multiplied by $dy$. Then

$$I_y = I_c + my^2 = 2\pi R^3 (r^2 - y^2)^{\frac{1}{2}}\, dy + 2\pi R (r^2 - y^2)^{\frac{3}{2}}\, dy + 4\pi R y^2 (r^2 - y^2)^{\frac{1}{2}}\, dy.$$




For the whole ring the formula becomes

$$I_w = \int_{-r}^{+r} [2\pi R^3 (r^2 - y^2)^{\frac{1}{2}} + 2\pi R (r^2 - y^2)^{\frac{3}{2}} + 4\pi R y^2 (r^2 - y^2)^{\frac{1}{2}}]\, dy = \pi^2 r^2 R [R^2 + \tfrac{5}{4} r^2] \ldots (1).$$

Since $2\pi^2 r^2 R$ = mass of ring, $M$, $I_w = \frac{M}{2}[R^2 + \frac{5}{4} r^2]$.

From (1), when $R = 0$, $I_w = 0$, the reversed moment of inertia of the inner portion of the ring exactly balancing that of the outer portion as the inner part backs over the center and becomes equal in mass to the outer part. This interesting hypothetical process by which the ring converts itself into two coinciding spheres with annulment of moment of inertia may perhaps be better appreciated by finding the moments of inertia of the outer and inner parts separately.

Take as the outer portion of the ring the part outside a cylindrical shell of radius $R$, with axis at $O$ and perpendicular to the plane $PQ$. The inner part referred to below will be within this cylinder.

As before, $I_o = \frac{\pi}{4}(A_1^4 - a_1^4)\, dy$, only in this case $a = R$.

$$\therefore I_o = \frac{\pi}{4}\{[R + (r^2 - y^2)^{\frac{1}{2}}]^4 - R^4\}\, dy.$$

As before, after making obvious changes,

$$my^2 = \pi y^2 [2R(r^2 - y^2)^{\frac{1}{2}} + r^2 - y^2]\, dy.$$

Then

$$I_{O_1} = I_o + my^2 = \frac{\pi}{4}\int_{-r}^{+r} [4R^3 (r^2 - y^2)^{\frac{1}{2}} + 6R^2 (r^2 - y^2) + 4R^2 (r^2 - y^2)^{\frac{3}{2}} + r^4 - 2r^2 y^2 + y^4]\, dy + \pi \int_{-r}^{+r} [2R(r^2 - y^2)^{\frac{1}{2}} + r^2 - y^2] y^2\, dy.$$

$$\therefore I_{O_1} = \frac{\pi}{2}[\pi r^2 R^3 + \tfrac{5}{4}\pi r^4 R + 4r^3 R^2 + \tfrac{16}{15} r^5]$$

$$= \frac{\pi^2 r^2 R}{2}[R^2 + \tfrac{5}{4} r^2] + [2\pi r^3 R^2 + \tfrac{8}{15}\pi r^5] \ldots (2).$$

It may be noted that (2) is equal to one-half the moment of inertia of the entire ring, plus the quantity $2\pi r^3 R^2 + \frac{8}{15}\pi r^5$. When, in this case, $R=0$, the outer portion evidently becomes a sphere of radius $r$, and $I_{O_1}$ becomes $\frac{8}{15}\pi r^5$, or $\frac{2}{5}Mr^2$, where $M$ is the mass of the resulting sphere. This, of course, is as it should be.

Similar computations for the inner portion of the ring give:

$$I_i = \frac{\pi^2 r^2 R}{2}[R^2 + \tfrac{5}{4} r^2] - [2\pi r^3 R^2 + \tfrac{8}{15}\pi r^5] \ldots (3).$$

When $R=0$, the inner portion of the ring backs across the center, "turning wrong side out" in becoming a sphere, and its moment of inertia is $-\frac{8}{15}\pi r^5$, or $-\frac{2}{5}Mr^2$. It may be noticed that, when $R=0$, the value of $I$ for the inner portion of the ring is the same as that for the outer portion, but with its sign changed. On comparing (2) and (3), it is seen that their sum is equal to (1) for $R>r$ and $R=r$, and that their sum is equal to zero when $R=0$. Between $R=r$ and $R=0$, equation (1) passes through a series of complications which are interesting, but perhaps more so to the mathematician than to the experimental physicist.

## DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

**365. Proposed by C. N. SCHMALL, New York City.**

**In still water, a steam tug goes 6 miles an hour less when towing a barge than when alone. Having drawn the barge 30 miles up a stream, whose current runs 1 mile an hour, it returns alone and completes the journey in 12 8/11 hours. Find the rate of the tug in still water.**

**Solution by MARY INGRAM, Lucus (Kansas) High School.**

Let $x$=rate of the tug in still water; $x-7$=rate up stream with barge; $x+1$=rate down stream.

$\frac{30}{x-7}$=time up stream with barge; $\frac{30}{x+1}$=time in returning down stream alone.

$$\frac{30}{x-7}+\frac{30}{x+1}=12\tfrac{8}{11}=\tfrac{140}{11},$$

or, clearing of fractions, $330x+330+330x-2310=140x^2-840x-980$.

Whence, $7x^2-75x+50=0$, and therefore $7x=70$ or 5.

$x=10$ or $\frac{5}{7}$. $\frac{5}{7}$ is not admissible.

Also solved by M. A. Muzzy, A. H. Holmes, J. Scheffer, and H. C. Feemster.

366. Proposed by WILLIAM HOOVER, Ph. D., Professor of Mathematics and Astronomy, Ohio University Athens, Ohio.

Eliminate $m$ from the equations

$$3a^2m^4+4aym^3+6axm^2-(x^2+y^2-4ax)=0;$$
$$(x^2+y^2-4ax)m^4-6axm^2+4aym-3a^2=0.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Dividing the first equation by $m^4$, and adding, we get

$$4ay(m+\frac{1}{m})-6ax(m^2-\frac{1}{m^2})+(x^2+y^2-4ax)\ (m^4-\frac{1}{m^4})$$

and dividing this by $m+\frac{1}{m}$, we have

$$4ay-6ax(m-\frac{1}{m})+(x^2+y^2-4ax)\ (m^2+\frac{1}{m^2})\ (m-\frac{1}{m})=0.$$

Putting $m-\frac{1}{m}=p$, the last expression reduces to

$$4ay-6axp+(x^2+y^2-4ax)\,(p^2+2)p=0\ldots(\mathrm{I}).$$

Again dividing both of the original equations by $m^2$ and adding, we have

$$3a^2(m^2-\frac{1}{m^2})+4ay(m+\frac{1}{m})+(x^2+y^2-4ax)\ (m^2-\frac{1}{m^2})=0.$$

Suppressing the factor $m+\frac{1}{m}$, we get

$3a^2(m-\frac{1}{m})+4ay+(x^2+y^2-4ax)(m-\frac{1}{m})=0$; or

$$3a^2p+4ay+(x^2+y^2-4ax)p=0\ldots(\text{II}).$$

From (II), we get $p=-\dfrac{4ay}{x^2+y^2-4ax+3a^2}$, and substituting this in (I), we get

$$(x^2+y^2+3a^2-4ax)^2(3a^2+10ax-x^2-y^2)-16a^2y^2(x^2+y^2-4ax)=0.$$

Also solved by M. A. Harding, S. Lefschetz, and A. H. Holmes.

367. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the simultaneous equations:

$$\frac{2x}{1+x^2}=y\ldots(1);\quad \frac{2y}{1+y^2}=z\ldots(2);\quad \frac{2z}{1+z^2}=u\ldots(3);\quad \frac{2u}{1+u^2}=x\ldots(4).$$

Solution by PROFESSOR F. L. GRIFFIN, Reed College, Portland, Oregon.

By inspection three solutions are $x=y=z=u=1$, or $-1$, or $0$; and there can be but fourteen others. Now let $x=i\tan\theta$, $[i=\sqrt{-1}]$, whence $y=i\tan 2\theta$, $z=i\tan 4\theta$, $u=i\tan 8\theta$, and $x=i\tan 16\theta$. But $\tan 16\theta=\tan\theta$ for finite values of $\theta$ only if $16\theta=\theta+n\pi$, or $\theta=n\pi/15$. Thus we have

$$\begin{aligned}
x&=0,\ i\tan\pi/15,\ i\tan 2\pi/15,\ i\tan 3\pi/15,\ \ldots,\ i\tan 14\pi/15;\\
y&=0,\ i\tan 2\pi/15,\ i\tan 4\pi/15,\ i\tan 6\pi/15,\ \ldots,\ i\tan 28\pi/15;\\
z&=0,\ i\tan 4\pi/15,\ i\tan 8\pi/15,\ i\tan 12\pi/15,\ \ldots,\ i\tan 56\pi/15;\\
u&=0,\ i\tan 8\pi/15,\ i\tan 16\pi/15,\ \ldots\ldots\ldots\ldots
\end{aligned}$$

the values for $y$, $z$, $u$ being of course the same sets as for $x$ in different orders. The values $x=+1$, $-1$ correspond to infinite values of $\theta$ for which $\tan\theta=-i$, $+i$.

## GEOMETRY.

393. Proposed by S. LEFSEHETZ, University of Nebraska.

Draw a triangle having a given angle, and with its vertices on three given concentric circles.

I. Solution by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Nebraska.

Let $x^2+y^2=c^2$, $x^2+y^2=b^2$, $x^2+y^2=a^2$, be the given circles, and $\phi$ the given angle. Place the vertex of the given angle, $\phi$, on the circumference of the first circle at 0, $c$, the angle being formed by the lines,

$$y=lx+c, \text{ and } y=\frac{l+\tan\phi}{1-l\tan\phi}x+c,$$

where $l$ is the tangent of the angle made by the first line with the $x$-axis, in the usual sense.

Substituting the equation of the first line in the equation of the second circle, the condition for intersection is

$$l \gtreqless \frac{\sqrt{(c^2-b^2)}}{b}, \text{ or } l \lesseqgtr -\frac{\sqrt{(c^2-b^2)}}{b},$$

and substituting the equation of the second line in the equation of the third circle, the condition for intersection is,

$$\frac{l+\tan\phi}{1-l\tan\phi} \gtreqless \frac{\sqrt{(c^2-a^2)}}{a}, \text{ or } \frac{l+\tan\phi}{1-l\tan\phi} \lesseqgtr -\frac{\sqrt{(c^2-a^2)}}{a},$$

which reduces to

$$l \gtreqless \frac{a\tan\phi-\sqrt{(c^2-a^2)}}{\sqrt{(c^2-a^2)}\tan\phi+a}, \text{ or } l \lesseqgtr \frac{a\tan\phi+\sqrt{(c^2-a^2)}}{a-\sqrt{(c^2-a^2)}\tan\phi}.$$

Taking $c>b$ and $c>a$, there is no solution if

$$\phi>\tan^{-1}\frac{b\sqrt{(c^2-a^2)}+a\sqrt{(c^2-b^2)}}{\sqrt{(c^2-a^2)}\sqrt{(c^2-b^2)}-ab},$$

with the vertex of $\phi$ on the circumference of the first circle. But any angle gives a solution by placing its vertex on the circumference of either of the other circles, by solving in a similar manner. With these exceptions, any value of $l$, except the limiting value, which gives only one solution, gives four solutions.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

With one angle given, there is an indefinite number of triangles, and consequently I will modify the problem to having the three angles given; or, in other words, to describe a triangle whose vertices are in the circumference of three given concentric circles similar to a given triangle.

On the radius $OA$ draw the triangle $OAD$ similar to the given one; draw $DC$ from $OA:OB=AD:DC$. Make $\angle AOB=\angle ADC$, connect $B$ with $C$, then $ABC$ will be the

required triangle. For, by construction, $\triangle ABC$ is similar to $\triangle AOB$, and, consequently, $\triangle ABC$, the vertices of which lie on the three circumferences, is similar to $\triangle OAD$.

Solved similarly by C. N. Schmall.

394. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Stroud, England.

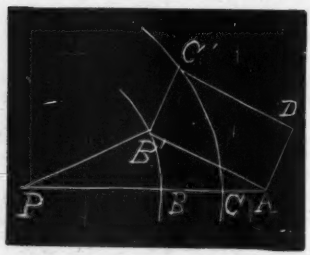
The joins of the excentres to the corresponding vertices of the pedal triangle are concurrent.

No solution of this problem has been received.

395. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

From a point $P$ without a rectangular field $ABC$ the distances $PA$, $PB$, $PC$ measured to the corners are respectively 70, 40, 60 chains. What is the area of the field?

Solution by A. H. HOLMES, Brunswick, Maine; J. SCHEFFER, A. M., Hagerstown, Maryland, and M. A. MUZZY, Waterville, Maine.




Let $P$ be the point without the field. Draw the line $PBA$; on it lay off $PB=40$ chains, $PC=60$ chains, and $PA=70$ chains. Then any point on the circumference of circle whose center is $P$ and radius $PB$ may be one corner of the rectangular field. One such field is $AB'C'D$. Hence, the problem is indeterminate. To make the problem determinate, one of the angles $APB'$, $APC'$, or $B'PC'$ should be given.

396. Proposed by DANIEL KRETH, Oxford, Iowa.

In the triangle $ABC$, $AB=214$, $BC=263$, and $AC=405$. A point $P$ is situated in the same horizontal plane; angle $BPA=13^\circ\ 30'$ and angle $BPC=29^\circ\ 50'$. Find the distances, $AP$, $BP$, and $CP$.

Solution by M. A. MUZZY, Waterville, Maine.

Draw a circle through $P$, $A$, $C$, cutting $BP$ in some point $D$. Then $BPA=ACD$, $BPC=CAD$, or $BPC=180^\circ-CAD$. Hence $CAD=29^\circ\ 50'$ or $150^\circ\ 10'$, $ADC=136^\circ\ 40'$ or $16^\circ\ 20$. There are four solutions.

In triangles $CAD$ find $CD_1=CD_2=293.60$, $CD_3=CD_4=716.43$.

In triangle $ABC$ find $ACB=28^\circ\ 23.8'$. $BCD=ACB\mp ACD$. Whence $BCD_1=BCD_4=14^\circ\ 53.8'$, $BCD_2=BCD_3=41^\circ\ 53.8'$.

In triangles $BCD$ find $CBD_1=105^\circ\ 21.2'$, $CBD_2=77^\circ\ 13.4'$, $CBD_3=119^\circ\ 27.8'$, $CBD_4=156^\circ\ 46.0'$.

In triangles $CBP$ find $BP_1=372.59$, $BP_2=389.09$, $BP_3=269.93$, $BP_4=422.58$; $CP_1=509.80$, $CP_2=515.58$, $CP_3=460.29$, $CP_4=208.55$.

In triangles $APB$ find $AP_1=166.77$, $AP_2=572.10$, $AP_3=57.96$, $AP_4=600.81$.

Also solved by J. Scheffer and H. C. Feemster.

## CALCULUS.

320. Proposed by J. F. LAWRENCE, Stillwater, Oklahoma.

Show that, if $u=1+A_1x+\frac{1}{2!}A_2x^2+\frac{1}{3!}A_3x^3+\ldots$ where the quantities $A$ are connected by the relation $A_m=mA_{m-1}-\frac{1}{2}(m-1)(m-2)A_{m-3}$, then $\log[u(1-x)^{\frac{1}{2}}]=\frac{1}{2}x+\frac{1}{4}x^2$. [From Forsyth's *Differential Equations*, p. 48.]

I. Solution by E. B. ESCOTT, Ann Arbor, Michigan.

$$u=1+A_1x+\frac{1}{2!}A_2x^2+\ldots+\frac{A_m}{m!}x^m+\ldots$$

where $A_m=mA_{m-1}-\frac{1}{2}(m-1)(m-2)A_{m-3}$.

Form the expression which has for the coefficient of $x^m$,

$$A_m-mA_{m-1}+\tfrac{1}{2}(m-1)(m-2)A_{m-3}$$

multiplied by some factor. Then such an expression will have only a finite number of terms. Such an expression is

$$u-xu+\tfrac{1}{2}\int x^2u\,dx;$$

also its derivative

$$(1-x)\frac{du}{dx}+(\tfrac{1}{2}x^2-1)u.$$

Therefore, we have for the differential equation of $u$,

$$(1-x)\frac{du}{dx}+(\tfrac{1}{2}x^2-1)u=0.$$

Separating variables,

$$\frac{du}{u}+\frac{\frac{1}{2}x^2-1}{1-x}dx=0.$$

whence

$$\log u(1-x)^{\frac{1}{2}}=\frac{x}{2}+\frac{x^2}{4},$$

the constant of integration being zero, since when $x=0$, $u=1$, and $\log 1=0$.

II. Solution by A. M. HARDING, University of Arkansas, Fayetteville, Arkansas.

From the given relation we have

$$A_1=1,\ A_2=2,\ \ldots,\ \tfrac{1}{2}\frac{A_{m-3}}{(m-3)!}=\frac{mA_{m-1}}{(m-1)!}-\frac{mA_m}{m!}\ldots(1).$$

Let $\phi(x)=(1-\frac{x^2}{2})(1+A_1x+\frac{A_2}{2!}x^2+\ldots+\frac{1}{m!}x^m+\ldots)(1+x+x^2+\ldots+x^m+\ldots)$

$$=[1+A_1x+(\frac{A_2}{2!}-1)x^2+\ldots+(\frac{A_m}{m!}-\tfrac{1}{2}\frac{A_{m-2}}{(m-2)!})x^m+\ldots]\times$$

$$[1+x+x^2+\ldots+x^m+\ldots]$$

Let us assume that both these series are absolutely convergent. Multiply them in the ordinary way and obtain a series $\Sigma c_mx^m$ where $c_m$, after reduction, has the value

$$c_m=\frac{A_m}{m!}+\frac{A_{m-1}}{(m-1)!}+\tfrac{1}{2}\Big[\frac{A_{m-2}}{(m-2)!}+\frac{A_{m-3}}{(m-3)!}+\ldots+\frac{A_2}{2!}+A_1+1\Big].$$

From the relation (1) we have

$$\begin{cases}\tfrac{1}{2}=\dfrac{3A_2}{2!}-\dfrac{3A_3}{3!}=1+A_1+\dfrac{A_2}{2!}-\dfrac{3A_3}{3!} & (\text{since } \dfrac{3A_2}{2!}=1+A_1+\dfrac{A_2}{2!}=3.\\[2ex] \tfrac{1}{2}A_1=\dfrac{4A_3}{3!}-\dfrac{4A_4}{4!}.\\[2ex] \tfrac{1}{2}\dfrac{A_2}{2!}=\dfrac{5A_4}{4!}-\dfrac{5A_5}{5!}.\\ \cdots\cdots\cdots\\ \tfrac{1}{2}\dfrac{A_{m-2}}{(m-2)!}=(m+1)\dfrac{A_m}{m!}-(m+1)\dfrac{A_{m+1}}{(m+1)!}.\end{cases}$$

Adding these equations, we obtain

$$0=c_m-(m+1)\frac{A_{m+1}}{(m+1)!}.$$

$$\therefore c_m=(m+1)\frac{A_{m+1}}{(m+1)!} \text{ and } \phi(x)=\Sigma c_m x^m=A_1+A_2x+\frac{1}{2!}A_3x^2+\ldots$$

$$+\frac{1}{m!}A_{m+1}x^m.$$

$$\therefore \phi(x)=\frac{du}{u}. \quad \text{But } \phi(x)\equiv(1-\frac{x^2}{2})(1-x)^{-1}.u=\frac{u}{2}[1+x+\frac{1}{1-x}].$$

$$\therefore \frac{du}{u}-\frac{dx}{2(1-x)}=(\tfrac{1}{2}+\frac{x}{2})dx. \quad \text{Integrating, we obtain}$$

$$\log u+\tfrac{1}{2}\log(1-x)=\frac{x}{2}+\frac{x^2}{4}+c.$$

Now when $x=0$, $u=1$, and hence $c=0$.

$$\therefore \log[u(1-x)^{\frac{1}{2}}]=\frac{x}{2}+\frac{x^2}{4}.$$

C. N. Schmall should have received credit for solving 317.

## MECHANICS.

357. **Proposed by W. J. GREENSTREET, M. A., Stroud, England.**

A portion of a circular cylinder cut off by two planes through the axis rests with its curved surface on two rough horizontal rails parallel to its axis, the coefficients of friction $\mu_1$, $\mu_2$ at upper and lower rails respectively. If the body is in limiting equilibrium at both rails when the plane through the axis and the center of gravity is perpendicular to both rails, find the distance of the center of gravity in terms of the distance betwen the rails, the inclination of their plane to the horizon, and the cofficients of friction.

358. **Proposed by W. J. GREENSTREET, M. A., Stroud, England.**

Two heavy particles connected by a string, length $l$, lie one on each of two inclined planes with common horizontal edge and of angles $\alpha$ and $\beta$. The inclination of the string to the edge varies as the inclination to the horizon of a simple pendulum of length $l(\sin\alpha+\sin\beta)$.

No solutions of these problems have been received.

259. **Proposed by J. SCHEFFER, A. M., Hagerstown, Md.**

A uniform beam of the weight $W$, rests on a horizontal plane, and leans against a vertical wall, but so as *not* to lie in a vertical plane. Denoting the pressure upon the horizontal and vertical planes, respectively, by $x$

and $y$, the coefficients of friction respectively, by $\mu$ and $\mu'$; the angle which the perpendiculars from the foot of the beam upon the intersection of both planes makes with the beam by $\phi$; the angle which this perpendicular makes with the direction of the friction peg by $\xi$; and the angle, which the projection of the beam upon the vertical wall makes with the vertical line, by $\psi$. To prove: $(1+\mu^2)\mu'^2\sin^4\psi-[1+\mu'^2+4\mu^2\mu'^2]\sin^2\psi+4\mu^2\mu'^2=0$; $\tan\xi=\mu'\cos\psi$, $\tan\phi=\mu'\cot\psi$, $x=\dfrac{\mu\cos\xi}{1+\mu\mu'\sin\psi\cos\xi}W$, $y=\dfrac{1}{1+\mu\mu'\sin\psi\cos\xi}W$.

Solution by S. G. BARTON, Ph. D., Swarthmore College.

Resolving the forces horizontally perpendicular to the line of intersection of the planes, and vertically, we have, respectively,

(1) $$x\,\mu\cos\xi=y$$

and $$x+\mu\,y\sin\psi=W.$$

Solving these simultaneous equations for $x$ and $y$ we have

(2) $$x=\frac{1}{1+\mu\,\mu'\sin\psi\cos\xi}W,\quad y=\frac{\mu\cos\xi}{1+\mu\,\mu'\sin\psi\cos\xi}W.$$

Resolving horizontally parallel to the line of intersection we have

$$y\,\mu'\cos\psi=x\,\mu\sin\xi,$$

which by use of (1) reduces to

(3) $$\mu'\cos\psi=\tan\xi.$$

The beam, its projection upon the horizontal plane, and the line from the foot of the beam perpendicular to the line of intersection of the planes are three edges of a trihedral angle, one dihedral angle of which is a right angle, and the face angles $\xi$, $\phi$, and $\beta$, where $\beta$ represents the inclination of the beam to the horizon. Considering this trihedral angle as bounded by a sphere with its center at the vertex, we have, from spherical trigonometry,

(4) $$\cos\beta=\cos\phi\,\sec\xi.$$

Considering the other end of the beam, the angle between the beam and its projection on the vertical plane is $90°-\phi$, and that between the beam and the vertical line through its end is $90°-\beta$. From this trihedral angle we find similarly

(5) $$\sin\beta=\cos\psi\,\sin\phi.$$

Squaring and adding these relations to eliminate $\beta$, we find

$$1=\sin^2\phi\cos^2\psi+\cos^2\phi\sec^2\xi,$$

and using (3) to reduce,

$$\sec^2\phi=\tan^2\phi\cos^2\psi+1+\mu'^2\cos^2\psi$$
$$\tan^2\phi(1-\cos^2\psi)=\mu'^2\cos^2\psi, \text{ or}$$
$$\tan\phi=\mu'\cot\psi. \tag{6}$$

Taking moments about the top of the beam in the vertical plane containing the beam, calling the length $2L$, we have,

$$2L\cos\beta\, x-2L\sin\beta\,\mu\, x-L\cos\beta\, W=0, \text{ or}$$
$$2(1-\mu\tan\beta)x-W=0;$$
or using (2), $$1-2\mu\tan\beta-\mu\mu'\sin\psi\cos\xi=0.$$

Dividing (5) by (4) and using (6), we find, $\tan\beta=\dfrac{\mu'\cos^2\psi}{\sin\psi}\cos\xi$, which substituted, gives $\sec\xi=\mu\mu'(2\dfrac{\cos^2\psi}{\sin\psi}+\sin\psi)$, or by (3),

$$\surd(\mu'^2\cos^2\psi+1)=\mu\mu'(\frac{2-\sin^2\psi}{\sin\psi}).$$

Squaring, we easily find

$$(1+\mu^2)\mu'^2\sin^4\psi-(1+\mu'^2+4\mu^2\mu'^2)\sin^2\psi+4\mu^2\mu'^2=0.$$

**260. Proposed by W. J. GREENSTREET, M. A., Stroud, England.**

To the ends of a fine inextensible string, length $2l$, are attached to equal, smooth, spherical, equally elastic ($e$) particles. At first the middle point of the string touches a rigid, fixed, circular rim, radius $a$, and the particles are $2l$ apart. They are now projected with equal velocities perpendicular to the string and curl around the rim. If $l$ is greater than $\pi a$, find the condition that the particles will move after collision along tangents to the rim, the whole motion being on a smooth horizontal plane.

**Solution by H. PRIME, Boston, Massachusetts.**

Let $P$ be the middle of the string, $Q$ the point of impact, $O$ the center of the rim, $R$ the point of tangency with the string at the moment of impact and $AB$ the normal through $Q$; $\theta$ and $\phi$ the angles which the velocities make

with $AB$ before and after impact; $u$ and $v$ the components of the velocities along $AB$ before and after impact. The tangential components of these velocities (along $QP$) before and after impact are $u\tan\theta$ and $v\tan\phi$, which are unchanged by the impact, since the contact is smooth. Hence it must be that $u\tan\theta=v\tan\phi$, or since $v=eu$, $\tan\theta=e\tan\phi$. Also since the string tangent to the rim is a normal to its involute, $\theta+\phi=90°$ and $\tan\phi=\cot\theta$. Hence $\tan\theta=1/e$. But $\theta=PQR$ and $POR=90°+\theta$. Therefore the required condition is $l=a(\pi/2+\tan^{-1}1/e+1/1/e)$.

Also solved by S. G. Barton.

## PROBLEMS FOR SOLUTION.

### ALGEBRA.

374. Proposed by H. PRIME, Boston, Massachusetts.

Divide an angle of 30° into two parts so that the product of the third and fourth powers of their sines (or cosines) shall be a maximum. To be solved without using the methods of calculus. [From *The Maine Farmers' Almanac*, 1912.]

375. Proposed by S. LEFSCHETZ, University of Nebraska.

Prove that $\frac{e}{2m+2}<e-(1+\frac{1}{m})^m<\frac{e}{2m+1}$. [Schlömilch.]

376. Proposed by W. W. BEMAN, Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

If $\frac{(1+1/m)^m}{e}=1-a_1\frac{1}{m}+a_2\frac{1}{m^2}-a_3\frac{1}{m^3}+\ldots$, prove $na_n=\sum_{k=1}^{k=n}\frac{k}{k+1}a_n-k$, and compute $a_1, a_2, a_3, \ldots, a_n$.

### GEOMETRY.

403. Proposed by C. E. GITHENS, Wheeling, West Virginia.

In a triangular field the sides enclosing an obtuse angle are 35 rods and 48 rods in length, Two straight lines are drawn from this vertex, and are at right angles to these sides. If these lines intersect the base 16 rods apart, how long is the third side of the field?

404. Proposed by S. LEFSCHETZ, Ph. D., University of Nebraska.

Let $ABC$ be a triangle, $M$ a point on $BC$, $BE$ the intersection of the perpendicular to $AM$ in $M$ with $AB$ and $AC$, $F$, the other intersection of the circle circumscribed to $ABC$ with the circle through $M$ and $A$ orthogonal to it in $A$. Prove that the points $C, D, E, F$ are on a circle, and find the envelope of the latter when $M$ describes $BC$.

405. Proposed by C. N. SCHMALL, New York City.

A plane cuts a constant volume from a given right cone. Prove that the minor axis of the section has a constant length.

## CALCULUS.

325. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Integrate $\int \frac{4x^6-a^3}{\sqrt{(x^6-a^3)}}\,dx.$

326. Proposed by C. N. SCHMALL, New York City.

Prove $\int_0^\infty \frac{[(\tan^{-1}ax)^2-(\tan^{-1}bx)^2]dx}{x}=\frac{1}{4}\pi^2\,(\log a-\log b).$

327. Proposed by RICHARD P. LOCHNER, 214 North 63rd Street, Philadelphia, Pennsylvania.

A hound is at the middle point of the side of a square field, and a fox is at an adjacent corner. How far will the hound run to catch the fox if the fox runs on the perimeter of the field and the hound runs directly towards the fox at all times, the hound running $n$ times as fast as the fox? Where will the race end?

328. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Prove that, $\frac{E_m}{\pi/2m}\int_0^{\pi/2m}[\sin\alpha+\sin(\frac{\pi}{m}+\alpha)+\ldots\sin(\pi\frac{m-1}{m}+\alpha)]\,d\alpha=\frac{2m\,E_m}{\pi}.$

[Sheldon and Hausmann: *Dynamo Electric Machinery*, Vol. I, p. 51.]

## MECHANICS.

269. Proposed by F. H. SAFFORD, Ph. D., University of Pennsylvania.

Three jointed rods, lengths 5, 2, and 5 units, are suspended from two pins which are 6 units apart and on a horizontal line, forming a freely jointed quadrilateral with the shortest side at the bottom. At the two lower joints are weights of $A$ and $2A$, respectively. Find the position of rest, the reactions along each bar, and the pressures on the pins.

270. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Burghfield, England.

A cycloid has its base vertical. Find the line of quickest descent from the middle point of the base, and its approximate inclination to the horizon.

## NUMBER THEORY AND DIOPHANTINE ANALYSIS.

186. Proposed by H. PRIME, Boston, Massachusetts.

Show that $\frac{(n+1)(n+2)\ldots(2n-2)}{(n-1)!}$ is an integer for all values of $n$.

## NOTES AND NEWS.

It is now hoped that the printing of the large Italian *Encyclopedia of Elementary Mathematics* will begin during the present year. The work is to be published by Hoepli of Milan. M.

The next meeting of the National Education Association will be held in Chicago, June 6-12, 1912. This will be the first time that this body has met in Chicago in twenty-five years. S.

The trustees of the University of Chicago have announced a system of retiring allowances which is to go into effect at once and for which endowment funds of more than two million dollars are to be set aside. S.

The ninth annual meeting of Ohio teachers of mathematics and science was held in Columbus on Friday and Saturday, March 29-30, 1912. Among the papers on mathematics was one on the teaching of trigonometry by Professor B. F. Yanney of Wooster University. S.

The next regular meeting of the Chicago Section of the American Mathematical Society will be held at Cleveland, Ohio, in affiliation with the American Association for the Advancement of Science. The latter will open its meeting on December 30, 1912. M.

According to the latest Annual Register of the American Mathematical Society about 50 of the 668 members are women. It is interesting to observe that the American Mathematical Society has a much larger per cent. of women members than the leading mathematical societies of Europe. According to the latest register of the German mathematical society (*Deutschen Mathematik Vereinigung*) only 5 of its 759 members are women; and only one of these 5 members is a German woman, while three of them are Americans and the remaining one is a Russian. The French mathematical society has also very few women members. The numbers of the women members of the Circolo Mathematico di Palermo and of the London Mathematical Society are considerably larger but they are much smaller than in our own society. M.

The February, 1912, number of the *Bulletin of the American Mathematical Society* contained a circular of 12 pages describing the French edition of the large mathematical encyclopedia. According to this circular 12 parts of the encyclopedia, covering 2762 pages, had appeared and 25 parts were then in press. A list of 160 contributors, including the names of such prominent French mathematicians as Poincaré, Picard, and Jordan, is given in this circular. More than one-third of the names appearing on this list are those of mathematicians residing in Paris, which gives some indication of the centralization of French Mathematics. It is also significant that this list contains the names of four Italian mathematicians. The circular is dated December, 1911, and it was issued by Gauthier-Villars of Paris, France. M.

C. F. GAUSS AND HIS SONS — A CORRECTION. In the January number of the MONTHLY, page 2, occurs the statement that John Bolyai was "a victim of the meanness of Gauss, as was also his own son who passed his life an exile here in Colorado."

My friend, Professor Cajori, informs me that no one of Gauss's four sons made his home in Colorado. He tells me that the son Eugen had a quarrel with his father, because of Eugen's wild life at the University of Göttingen, but when Eugen left home to sail for America, his father followed him, urged him to return home, and, when failing in this effort, offered him money to take with him to America. Eugen left home and came to this country by his own choice and against the wishes of his parents. His father was greatly grieved, as is shown by his correspondence. See the article "Carl Friedrich Gauss and his Children," in *Science*, N. S. Vol. 9, 1899, pages 697-704. G. B. HALSTED.

The thirtieth regular meeting of the Chicago Section of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 5 and 6, 1912. The total attendance upon the four half-day sessions was seventy-five, including fifty-six members of the Society.

There were twenty-nine papers presented, as follows:

(1) On birational transformations of three-space related to four-space varieties. (15 minutes.) Dr. S. Lefschetz, University of Nebraska.

(2) Optical interpretations in higher geodesy. (10 minutes.) Professor W. H. Roever, Washington University.

(3) Mechanisms for illustrating lines of force. (10 minutes.) Professor W. H. Roever.

(4) Deviations of falling bodies for a distribution not of revolution. Second paper. (15 minutes.) Professor W. H. Roever.

(5) The geometry of conformal rational transformations in a plane. (20 minutes.) Professor Arnold Emch, University of Illinois.

(6) The present state of the theory of Jupiter's five minor satellites. (20 minutes.) Professor Kurt Laves, University of Chicago.

(7) An extension of Descarte's rule of signs. Second paper. (20 minutes.) Professor D. R. Curtiss, Northwestern University.

(8) Equality in geometry. (15 minutes.) Professor J. K. Whittemore, Western Reserve University.

(9) On the relation between the empirical and the inertial trihedrons of gravitational astronomy. (20 minutes.) Dr. G. O. James, Washington University.

(10) A forgotten theorem of Newton's on planetary motion and an instrumental solution of Kepler's equation. (10 minutes.) Professor E. J. Wilczynski, University of Chicago.

(11) The relative number field $K(\sqrt[l]{a})$. (15 minutes.) Dr. G. E. Wahlin, University of Illinois.

(12) Analytic curves in noneuclidean space. Second paper. (20 minutes.) Dr. E. G. Bill, Purdue University.

(13) Algebra in the Quadripartitum numerorum of Johannes de Muris. (10 minutes.) Professor L. C. Karpinski, University of Michigan.

(14) Infinite systems of indivisible groups. (15 minutes.) Professor G. A. Miller, University of Illinois.

(15) Projective differential geometry of developable surfaces. (10 minutes.) Mr. W. W. Denton, University of Illinois.

(16) The method of monodromie and its application to three-parameter quartics. (10 minutes.) Professor R. P. Baker, University of Iowa.

(17) On Transcendentally transcendental functions. (25 minutes.) Professor R. D. Carmichael, Indiana University.

(18) On the theory of linear difference equations. (20 minutes.) Professor R. D. Carmichael.

(19) A general formula for the valuation of securities. (15 minutes.) Professor J. W. Glover, University of Michigan.

(20) On ordinary plane and skew curves. (By title.) Dr. E. L. Dodd, University of Texas.

(21) On the Spanish symbol $U$ for thousands. (By title.) Professor Florian Cajori, Colorado College.

(22) Infinite developments and the composition property $(K_{1,2}\, B_1)_*$ in general analysis. Mr. E. W. Chittenden, University of Chicago.

(23) Note on Mersenne's numbers. Mr. V. M. Spunar, Chicago, Ill.

(24) The projective differential geometry of three-spreads generated by $\infty^1$ planes in five dimensional space. Dr. E. B. Stouffer, University of Illinois.

(25) Multiplicative interrelations of certain classes of sequences of positive terms. Professor E. H. Moore, University of Chicago.

(26) On a theorem of Fejer's and an analogons to Gibbs' phenomenon. Dr. T. H. Gronwall, Chicago, Ill.

(27) Some asymptotic expressions in the theory of numbers. Dr. Gronwall.

(28) Closed orbits of ejection and related periodic orbits in the problem of three bodies. Professor F. R. Moulton, University of Chicago.

(29) Necessary and sufficient conditions for the interchange of limit and summation for a special type of series. Dr. T. H. Hildebrandt, University of Michigan.

An interesting feature of this meeting was the provision in the program for "notes and queries" on any topics not directly related to any of the papers presented. This brought out some very interesting informal discussion. S.

## BOOKS.

*A Treatise on the Analytical Geometry in Three Dimensions.* By George Salmon, D. D., D. C. L., LL. D., F. R. S., late Provost of Trinity College. Fifth edition. Revised by Reginald A. P. Rogers, Fellow of Trinity College, Dublin. Vol. I. 8vo, cloth. xxii+470 pages. Price, $3.00. New York: Longmans, Green & Co.

In the revision of this, the most exhaustive and noted work of its kind in the English language, it is the purpose of Mr. Rogers to preserve the substance of the fourth edition. Copies of the fourth and last edition had become quite difficult to obtain. In this new edition, Mr. Rogers has added some new mattter of much value. For example, in Chapter V, we find excellent illustrations of models of the different species of quadrics, articles or paragraphs on the analytical classification of real quadrics ($88_a$), and elsewhere, on projection and Fiedler's projective coördinates ($144_c$), on non-Euclidean theory of distance and angle ($144_a$), and on the expression of twisted cubics and quartics by rational or elliptic parameters, ($333_a$, $347_a$, 348, 349).

In differential geometry, the aim has been to form a closer connecting link between Dr. Salmon's book and the more extensive and more purely analytical methods used by Bianchi, Darboux, and others. A number of other additions and improvements are made, but space forbids noting any more.

It is the purpose to bring out this fifth edition in two volumes. The fourth edition contained 612 pages. The type in this edition is much superior to that in the fourth edition. It is to be hoped that Volume II will soon be issued. F.

*The Elements of Statistical Methods.* By Wilford I. King, M. A., Instructor in Statistics in the University of Wisconsin. 8vo, cloth, xvi+250 pages. Price, $1.50. New York: The Macmillan Co.

So far as we know, the text before us is the first of its kind published in America. There have been texts published heretofore on statistical methods in Biology and perhaps other subjects, but none dealing with the method in general.

In the present work, the author has furnished those students of economics, sociology, and others of the educated public unfamiliar with the language and use of mathematics, a simple text easily within their grasp. The work is one that will prove of great value to the student of sociology, economics, as well as to those of some of the natural sciences. F.

*Introduction to Analytical Mechanics.* By Alex. Ziwet, Professor of Mathematics in the University of Michigan, and Peter Field, Ph. D., Assistant Professor of Mathematics in the University of Michigan. 8vo, cloth, ix+378 pages. Price, $1.60 net. New York: The Macmillan Co.

This volume is intended as a brief introduction to the study of mechanics for Junior and Senior students in colleges and universities. While the work is based largely upon Professor Ziwet's excellent Theoretical Mechanics, yet it differs from it in several essential respects. For example, the applications to engineering are omitted and the analytical treatment has been broadened. Geometrical ideas are made to precede analysis, and in so doing the idea of the vector is freely used while omitting at the same time the special methods and notations of vector analysis.

Those teachers who have used Professor Ziwet's larger work will welcome this briefer course by Ziwet and Field as more suitable for them if they have only a very limited time at their disposal to devote to the subject. F.

*The Teaching of Physics for Purposes of General Education.* By C. Riborg Mann, Associate Professor of Physics, The University of Chicago. 8vo, cloth, xxv+304 pages. Price, $1.25 net. New York: The Macmillan Co.

This is one of the Teachers' Library Series and the editor, President Butler of Columbia University, in his introduction sets forth eight fundamental principles which should be assumed in the teaching of Physics. Among them are: That the topics chosen and the method pursued should be determined for the intellectual needs and interests of the school; that teacher should put out of his mind the thought that each pupil before him is aiming to become a specialist in physical science; that physical science should not be something fixed and definite; that the student should know something of the men whose names are epoch-making in its history and development; that the ordinary standards for measuring time, space, weight and other characteristics should not be taken for granted; and that accurate measuring, while necessary and important, should hold not first place, but a subordinate place.

The author, Professor Mann, is not only an expert physicist, but he is an excellent teacher as well. He knows just how much help the student needs in making experiments in order to keep up his courage and his interest. His suggestions, therefore, as set forth in this book have a double value since they come from one who is both a teacher and an investigator.

The book is divided into three parts. The first part discusses, The Development of the Present Situation; the second part, Physics and Democratic Education; the third part, Hints at Practical Applications. F.

. *A Mental Arithmetic for Oral Review.* By John Brookie Faught, Ph. D., Professor of Mathematics in the Western State Normal School, Kalamazoo, Mich. Paper back, 24 pages. Published by the author.

It seems that the author of this little work is old-fashioned enough to wish to return to the good old days of the mental arithmetic. What will modern educators think of a man who is now writing a mental arithmetic when our school programs have long passed that stage in educational evolution? Yet we, too, are old-fashioned enough to wish that some of our college Freshmen had some of this mental arithmetic training. Not long ago we witnessed a college Senior in all seriousness add 1/2 and 2/3 and get 3/5 as a result. Now you college teachers of mathematics do not hold your noses while you read that statement, for we 'll warrant that many of you who read it have witnessed examples of the same stolid stupidity. There is something radically wrong somewhere, and to our mind, our critics are failing to put their fingers on the right spot. It seems to us that there is too much of the pyrotechnic display in our scheme of education, too morbid a desire for the novel and spectacular and too little of the serious purposes of life. The average college or university diplomas are coming to have about as much value educationally as a new national bank note would have commercially without the denomination stamp upon it. F.

*Linear Polars in the k-Hedron in n-Space.* A Dessertation to the Faculty of the Ogden Graduate School of Science, The University of Chicago, in Candidacy for the Degree of Doctor of Philosophy. By Harris Franklin MacNeish.

Copies of this thesis may be secured by sending 25 cents to the University of Chicago Press. F.

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## NOTE ON THE DEFINITION OF AN ASYMPTOTE.

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In teaching the subject of asymptotes to a class of sophomore students recently, my attention was called to a point that seemed worth noting, concerning certain definitions of an asymptote to a curve. Text-books in common use differ considerably in their definitions of an asymptote. There are three different definitions that I find by an examination of text-books.

Judging from these books, it seems that a definition often given is the following:

(1) If the tangent to a curve approaches a limiting position, as the distance of the point of contact from the origin is indefinitely increased, this limiting position is called an asymptote. More briefly, it is sometimes stated that an asymptote to a curve is a tangent line whose point of contact is at infinity, but such that the line is not entirely at infinity.

A second definition in rather common use is the following:

(2) An asymptote to a curve is a straight line whose distance from a point on the curve diminishes indefinitely as the point moves along the curve to an infinite distance from the origin.

A third definition sometimes given is:

(3) If a straight line cuts a curve at two points at an infinite distance from the origin, but is not entirely at infinity, the line is called an asymptote.

The book that I am using in the class referred to above, gives the first of these three definitions. It is then shown how to determine the oblique asymptotes of the curve

$$f(x, y)=0, \quad f(x, y) \text{ being a polynomial of degree } n,$$

by selecting $m$ and $b$ in the straight line

$$y=mx+b,$$